1. Let a, b, and c be three positive integers such that $a < b < c$ and $a + b + c = 12$. What is the largest possible value of c?

   (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

2. What is the value of $\sqrt{\dfrac{1}{2^6} + \dfrac{1}{6^2}}$?

   (A) $\frac{1}{10}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) $\frac{5}{24}$ (E) $\frac{7}{24}$

3. If $\frac{h}{x} = 12$, $\frac{h}{y} = 8$, and $x + y = 5$, then h equals

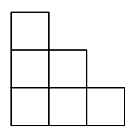
   (A) 4 (B) 12 (C) 15 (D) 24 (E) 48

4. Three distinct vertices of a regular hexagon are chosen at random. What is the probability that the three chosen vertices form an equilateral triangle?

   (A) $\dfrac{3}{20}$ (B) $\dfrac{1}{10}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{5}$ (E) $\dfrac{2}{5}$

5. Roger's assignment is to choose at least one of the six small squares in the figure below and color the chosen square(s) black, so that the resulting figure has an axis of symmetry. In how many different ways can Roger do this?

   (A) 8 (B) 15 (C) 16 (D) 21 (E) 32

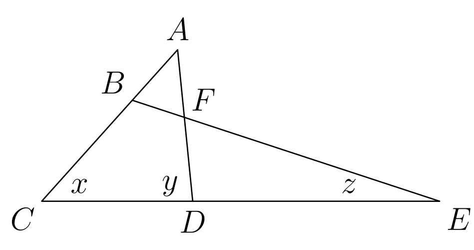


6. The number of positive integer factors of 108 that are even is

   (A) 4 (B) 6 (C) 8 (D) 12 (E) 16

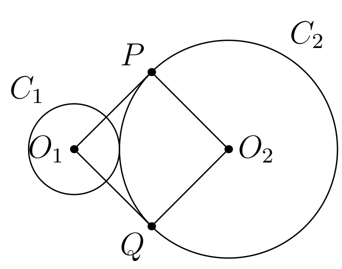
7. In the figure below, $AB = AF$. Which of the following expressions gives angle z in terms of x and y?

   (A) $\frac{y-x}{2}$ (B) $y - \frac{x}{2}$ (C) $\frac{y-x}{3}$ (D) $y - \frac{x}{3}$ (E) $y - x$



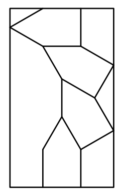
8. Circles C_1 and C_2 are externally tangent. Circle C_1 has center O_1 and radius 3. Circle C_2 has center O_2 and radius 12. The tangents from O_1 to circle C_2 are O_1P and O_1Q. What is the area of quadrilateral O_1PO_2Q?

   (A) 81 (B) 108 (C) 144 (D) 162 (E) 225



9. Erich finds the sum of the digits in every 8-digit number. The sum that occurs most often is
   (A) both 27 and 28 (B) 32 (C) both 36 and 37 (D) 39 (E) 41

10. A pentagonal tile is formed by joining an equilateral triangle and square with the same side length. Four such pentagonal tiles are placed inside a rectangle as shown. What is the ratio of the longer side of the rectangle to the shorter side?
   (A) $\sqrt{3} : 1$ (B) $2 : 1$ (C) $\sqrt{2} : 1$ (D) $3 : 2$ (E) $4 : \sqrt{3}$



11. What is the greatest number of lines that can be drawn in the plane such that each line intersects exactly four of the other lines?
   (A) 5 (B) 8 (C) 10 (D) 16 (E) Infinitely many

12. The decimal representation of the positive integer N consists of 2011 2's, followed by the digit d, followed by 2011 5's:
$$N = \underbrace{22\ldots2}_{\text{2011 2's}}\,d\,\underbrace{55\ldots5}_{\text{2011 5's}}.$$
If N is divisible by 7, then what is a possible value for the digit d?
   (A) 3 (B) 4 (C) 6 (D) 8 (E) 9

13. Which of the following lines is an axis of symmetry of the graph of $y = 2x + \frac{1}{x}$?
   (A) $y = (2+\sqrt{3})x$ (B) $y = 4x$ (C) $y = (2+\sqrt{5})x$
   (D) $y = (3+\sqrt{2})x$ (E) none of these

14. What is the sum
$$\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{2011\cdot2012\cdot2013}?$$
   (A) $\dfrac{2\cdot2012}{3\cdot2011\cdot2013}$ (B) $\dfrac{1}{3} - \dfrac{1}{3\cdot2013}$ (C) $\dfrac{1}{4} - \dfrac{1}{2012^2}$
   (D) $\dfrac{1}{3} - \dfrac{1}{3\cdot2012\cdot2013}$ (E) $\dfrac{1}{4} - \dfrac{1}{2\cdot2012\cdot2013}$

15. Let
$$p(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$
be a polynomial such that $p(1) = 1$, $p(2) = 2$, $p(3) = 3$, $p(4) = 4$, $p(5) = 5$, and $p(6) = 6$. What is $p(7)$?
   (A) 7 (B) 8 (C) 42 (D) 727 (E) $7^6 - 7$

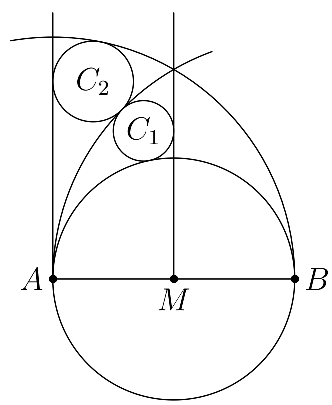
1. Let $ABCD$ be a trapezoid, with bases $AB = 4$ and $CD = 5$. Let P be the intersection of diagonals AC and BD, and let X and Y be points on sides AD and BC, respectively, so that XY passes through P and is parallel to AB. What is the length of XY?

2. If x is a root of $x^6 - 10x + 9 = 0$ and $x \neq 1$, then what is $x + x^2 + x^3 + x^4 + x^5$?

3. Find all triples (x, y, z) of real numbers such that $x + y = 2$ and $xy - z^2 = 1$.

4. Find the number of ways of arranging the letters A, B, C, D, E, F, and G, such that A appears before B, C appears before D, D appears before E, and F appears before G. For example, FCDAGBE is an acceptable arrangement.

5. Let A and B be two points in the plane, and let M be the midpoint of AB. We draw the circle with diameter AB, the circle centered at A with radius AB, and the circle centered at B with radius AB. We also draw the the line through A perpendicular to AB, and the line through M perpendicular to AB. We inscribe circles C_1 and C_2 in the regions shown.



Let r_1 and r_2 be the radii of circles C_1 and C_2, respectively. Find r_1/r_2.

6. Let S denote the set of sequences $(x_1, x_2, \ldots, x_{10})$, where each term is 0 or 1. Let

$$N = x_1x_2 + x_3x_4 + x_5x_6 + x_7x_8 + x_9x_{10}.$$

Let a be the number of sequences in S where N is odd, and let b be the number of sequences in S where N is even. Find a/b. (Express your answer in lowest terms.)

7. The sequence a_0, a_1, a_2, $\ldots$ is defined by $a_0 = 0$ and

$$a_n = \begin{cases} a_{n-1} + 1 & \text{if } n \text{ is odd,} \\ 3a_{n/2} & \text{if } n \text{ is even.} \end{cases}$$

Find the number of nonnegative integers n such that $a_n \leq 2011$.

8. Let $ABCD$ be a convex quadrilateral with an incircle Γ. (Each side of quadrilateral $ABCD$ is tangent to circle Γ.) Let I be the center of Γ. If $AB = 42$, $BC = 15$, $CD = 8$, and $DA = 35$, then find

$$IA \cdot IC + IB \cdot ID.$$